Capital Adequacy and Allocation: The Current State of Play

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Two perspectives…

Theory

- Theory = model
- "All models are wrong"
- Models must be wrong (an approximation) to be useful
- Model must capture an important part of reality
- **Property cat risk**

Practice

- Missing theory, no model, or
- Model can't be parameterized, or
- Model misses more than it captures
- Qualitative vs. quantitative
- **Multi-year business realities**
- **Casualty**

Section 1: Theory

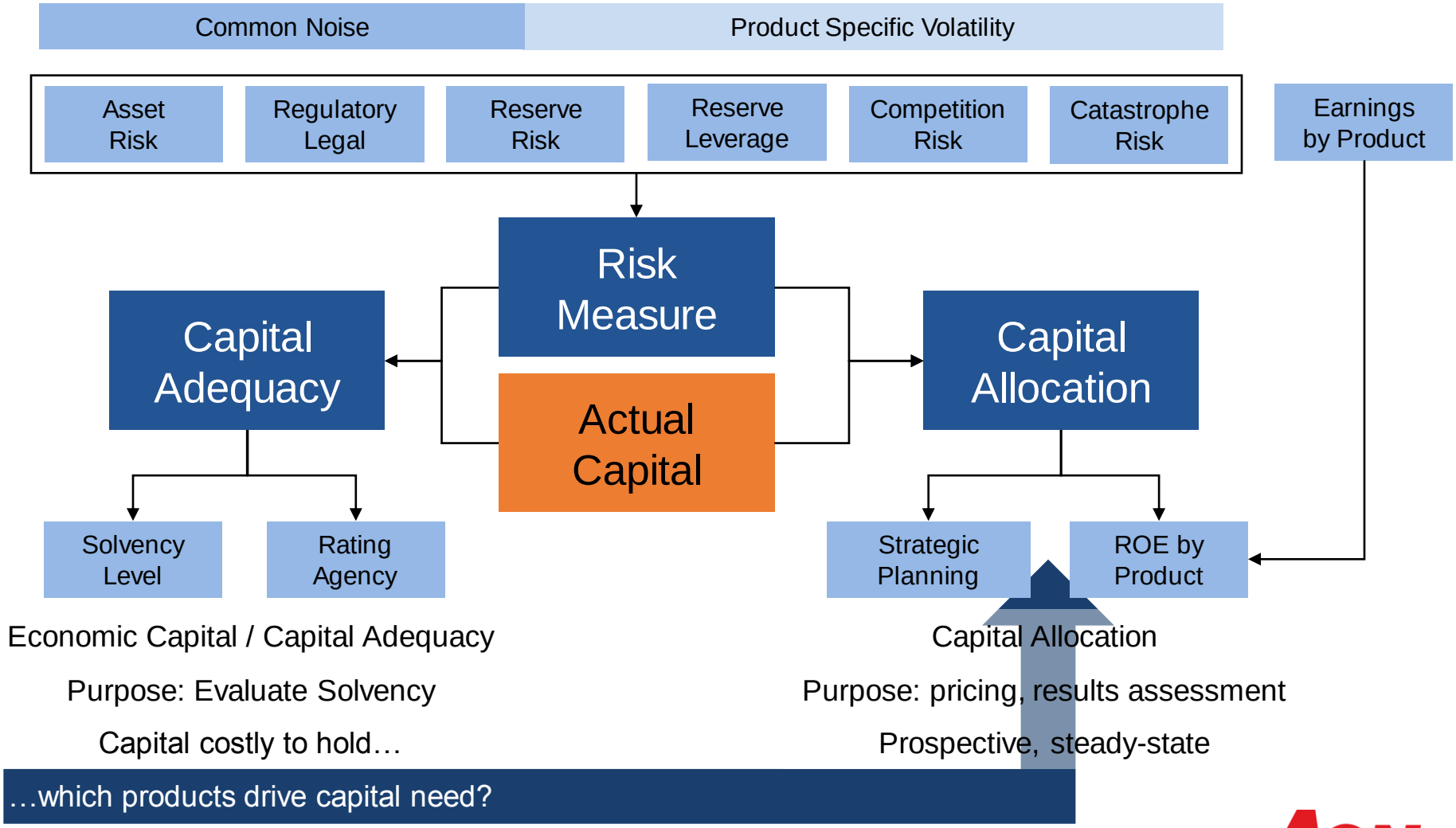
Capital

Accountants **determine** capital

Actuaries, and others, opine on its **adequacy**

Certain capital determinations have economic meaning because they trigger real world consequences: insolvency, regulatory supervision, etc.

Capital: adequacy and allocation



Capital adequacy is assessed by a risk measure

- Risk measures have one or more free parameters, quantifying safety margin
- Parameter allows risk measure to **describe** capital adequacy
- “Actual capital held provides 99.2% chance of covering losses over a one year horizon”
- BCAR = 337%
- Premium to surplus ratio = 0.75:1 (not risk based)

Risk of what?

- Risk measures apply to a random quantity

- Which is appropriate quantity to measure?
  - Losses
  - Total cash flow
  - Calendar year income
  - Accident year income
  - Comprehensive income
  - Value created
  - Market capitalization
  - When do you feel pain? =Below plan

- Should cash flows be nominal or discounted?
  - What discount rate should be used for discounting?

What is risk?

- Process vs. parameter risk vs. Knightian uncertainty
  - Reluctance to admit, with Keynes, that “We simply don’t know”

- How large a risk?
  - 100M households in US
  - $1M loss = 1¢ per household
  - $100M loss = $1 per household
  - $1B loss = $10 per household
  - $100B loss = $1000 per household
  - $1T loss = $10,000 per household

- Risk characteristics
  - Known loss agent
  - Unknown loss agent
  - Frequency of loss
  - Size of market
  - Certainty of loss estimate
  - Return on intellectual investment to understand risk

- Heterogeneous distribution of wealth and relationship to risk distribution
  - Ultimate risk bearers are individual insureds acting as investors
  - Population concentrations correlated to risk

What is risk? Different stakeholder perspectives

- Different perspectives drive different concerns
  - Shareholder: long/short-term economics, solvency (up to tail)
  - Debt holder: long/short-term economics, solvency (beyond tail)
  - Regulator: solvency, and loss given default (beyond tail)
  - Rating Agency: solvency
  - Policyholder: solvency vs. price
  - Employee: …

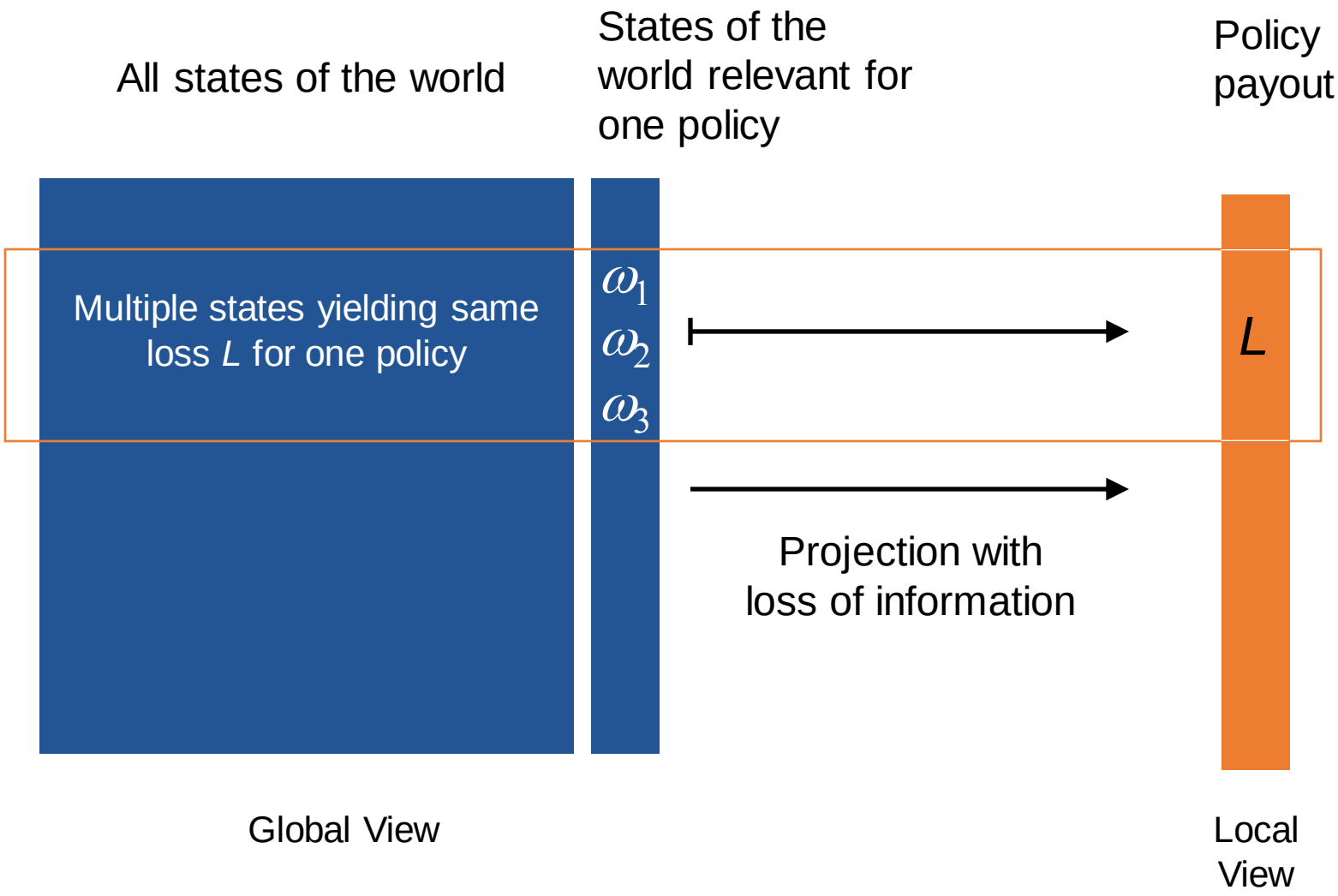
What is risk? A practical theoretical interpretation

- Rothschild-Stiglitz offer four possible definitions of when X is “more risky” than Y
  1. X = Y + noise
  2. Every risk averter prefers Y to X (utility)
  3. X has more weight in the tails
  4. Var(X) > Var(Y)

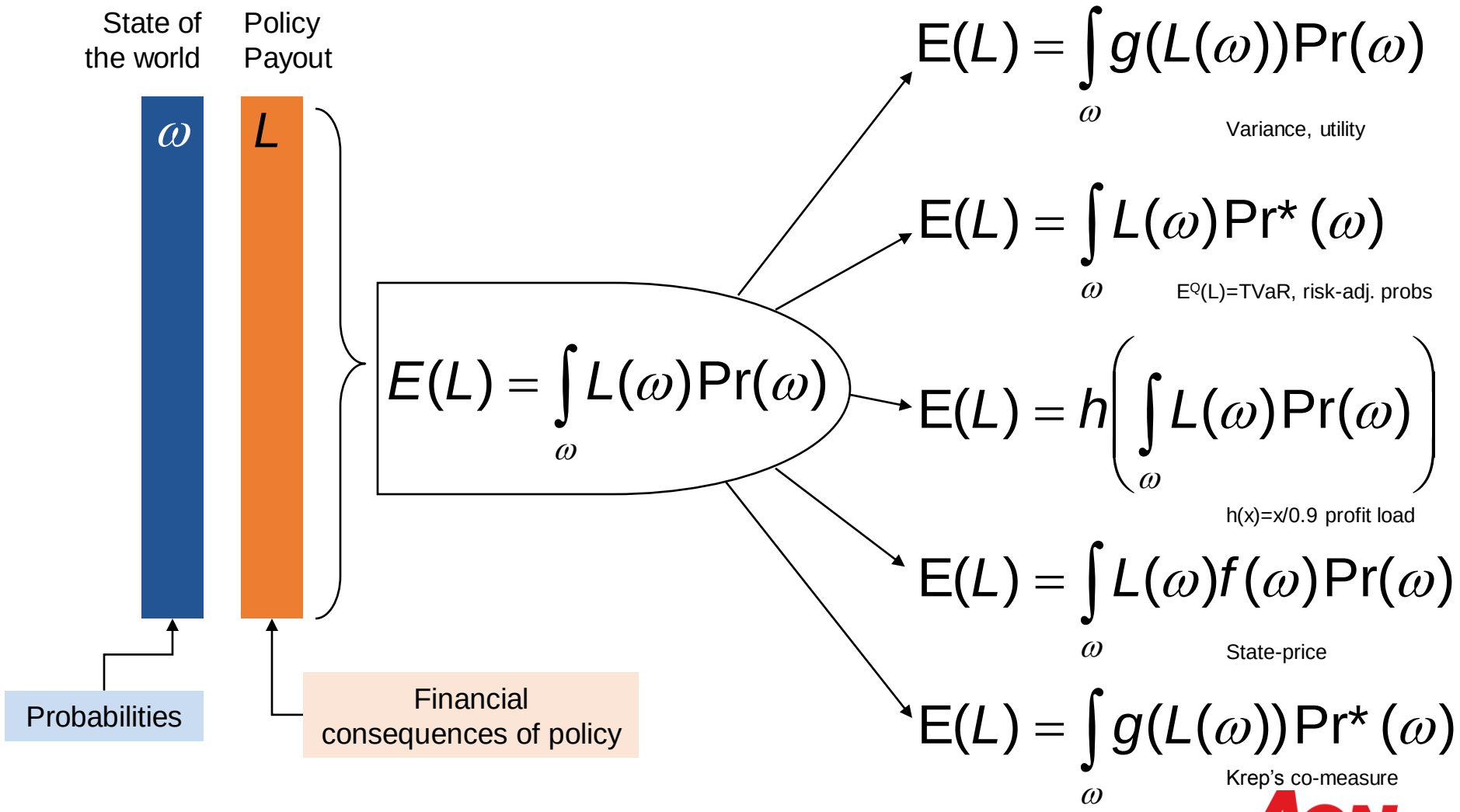
1, 2 & 3 are equivalent & are different from 4

- Problems collapsing a whole distribution to a single number
  - All moments may not be enough to determine distribution
  - “Local” vs. “global” views
  - Local = distribution based
  - Global = loss within broader economic context

Local and global views of risk



Families of risk measures



Important fundamental difference

Risk adjusted probabilities apply to **event probabilities**

Utilities apply to **outcomes**

Risk adjusted probabilities can **differentiate between equal loss outcomes**

Risk measures allowing for ignorance and uncertainty

- $\text{TVaR}_a(X) = \sup_{\{Q \text{ in } Ma\}} E_Q(X)$, where Ma = probability measures Q satisfying
  - $P(A)=0$ implies $Q(A)=0$
  - $dQ/dP < 1/a$ put all weight in the tail
  - Note: dP means $f(x)dx$ and dQ means $g(x)dx$, so $dQ/dP = g(x) / f(x)$

- $\text{Risk}(X) = \sup_{\{Q \text{ in } \boldsymbol{Q}\}} E_Q(X) - r(Q)$, where $r(Q)$ measures likelihood of Q
  - E.g. $r(Q) = E_Q[\log(dQ/dP)]$, is relative entropy

- State price density and covariance

$$E_Q(X) - E_P(X) = E_P(X dQ/dP) - E_P(X) = \text{Cov}(X, dQ/dP)$$

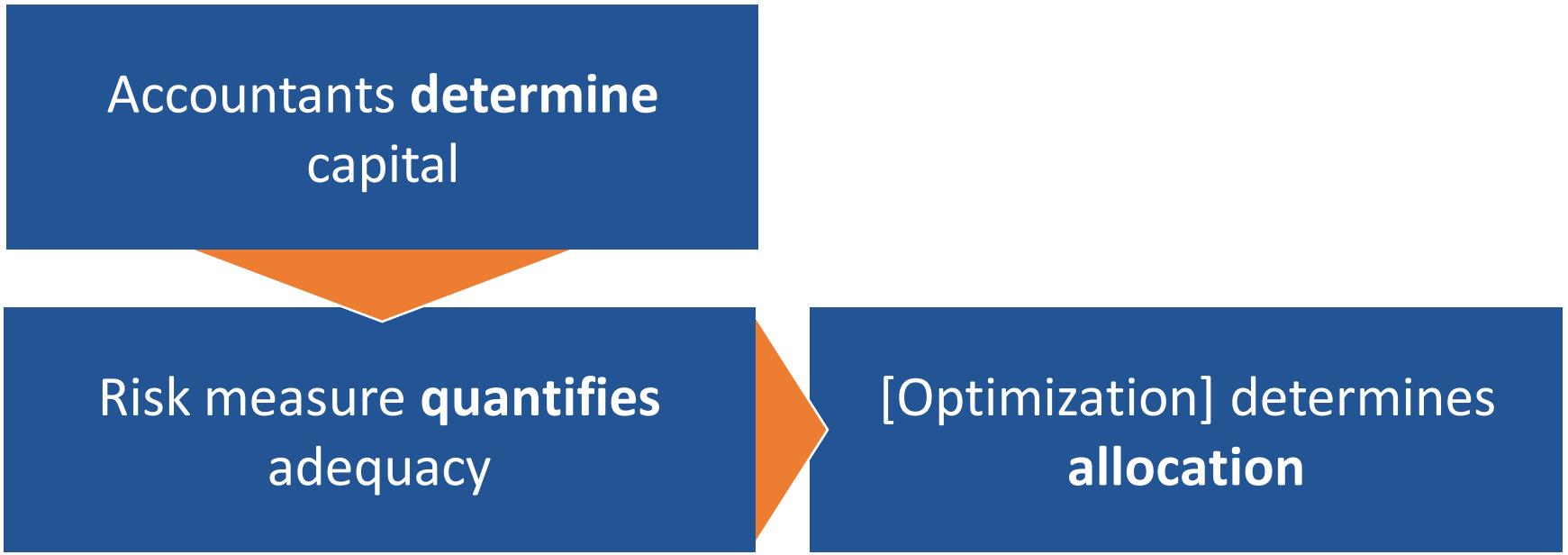
Before discussing how to allocate capital, ask “Why?”

- Capital is costly to hold
  - Double taxation
  - Agency costs
  - Credit sensitive customers
  - Skew averse investors
  - Capital market inefficiencies (costly to raise capital post-event)

- Proxy for allocation of cost of capital

- Cost must be allocated in order to effectively
  - Determine pricing
  - Assess BU profitability
  - Strategic planning

Risk measure assesses capital, drives allocation



…assuming adequacy remains unchanged

Inexorably led to Lagrangian constrained optimization

- Risks X_i priced with profits π_i
- Capital constraint k
- Risk measure r drives capital requirement
- **Select shares w_i to maximize $\Sigma\ \pi_i\ w_i$ subject to $r(\ \Sigma\ w_i\ X_i\ ) < k$**
- Introduce Lagrangian multiplier λ and the Lagrangian L

$$L = \Sigma\ \pi_i\ w_i - \lambda\ \{\ r(\ \Sigma\ w_i\ X_i\ ) - k\ \}$$

- To solve, differentiate wrt to w_i and λ and set equal to zero to get gradient, marginal risk = marginal return, pricing

$$\pi_i = \lambda\ \partial r\ /\ \partial w_i$$

- Links pricing with the risk measure and capital allocation through a cost of capital argument

Other capital allocations from capital adequacy measures

- Optimized via Lagrangian often actually a constrained optimization, Karush-Kuhn-Tucker (KKT) conditions
- Natural = E_Q, co[nditional]-measures, default put
- Diversification Index = $r(X) / \Sigma_i r(X_i)$ = peanut butter spread
- Magically additive = Euler's theorem
- Minimize claim on other areas of firm = equal risk VaR
- Gradient of risk measure reflecting insured's economic reality (Zanjani)
- Any allocation must pass fairness tests
  - No under-cut: can't allocate more than stand-alone capital
  - To regulator: too much diversification benefit
  - To children or grandchildren?
- Properties of risk measure translate into these properties of allocation

- Allocation should have an economic meaning

Capital allocation methods in ReMetrica

▪ VaR	Stand-alone VaR by unit
▪ Scaled VaR	Stand-alone VaR scaled to Total VaR
▪ Equal Risk VaR	Stand-alone VaR at adjusted percentile to sum
▪ TVaR	Stand-alone TVaR by unit
▪ Scaled TVaR	Stand-alone TVaR scaled to Total TVaR
▪ Equal Risk TVaR	Stand-alone TVaR at adjusted percentile to sum
▪ Phillips TVaR	$E(X_i \mid X > a)$, X_i = BU Loss, $X = X_1 + \ldots X_n$
▪ Merton-Perold VaR	VaR(whole book) – VaR(book excluding unit)
▪ Shapley VaR	Shapley value computed using VaRs
▪ Merton-Perold TVaR	TVaR(whole book) – TVaR(book excluding unit)
▪ Shapley TVaR	Shapley value computed using TVaRs, (Co-TVaR)
▪ Average of Methods	…a traditional actuarial approach
▪ Risk Adjusted Prob.	$E_Q(X) – E_P(X)$, Q=Aon transform, P=objective probability
▪ Risk Adjusted Prob.	$E_Q(X) – E_P(X)$, Q=Wang normal or T-normal transform
▪ Risk Adjusted Prob.	$E_Q(X) – E_P(X)$, Q=proportional hazard transform
▪ Covariance	Use CAPM-like proportion of variance by line scaled to total
▪ Percentile Layer	Loss weighted risk adjusted probability transform

Capital allocation method comparison

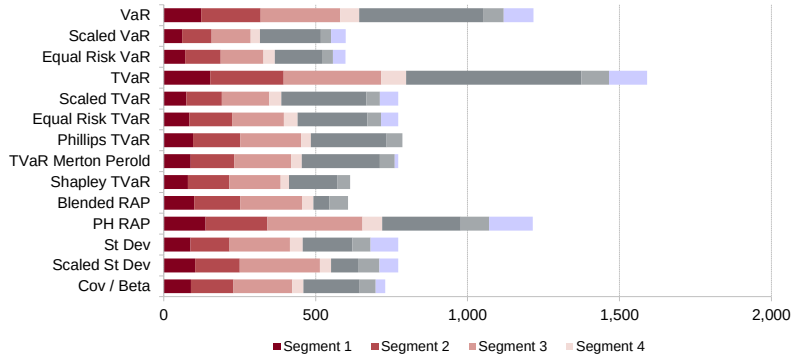
Percentile Threshold	0.40%

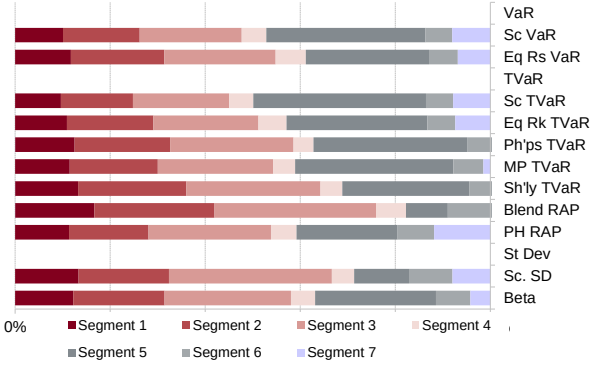
Business Unit	VaR	Scaled VaR	Equal Risk VaR	TVaR	Scaled TVaR	Equal Risk TVaR	Phillips TVaR
Segment 1	123	60	70	153	74	84	96
Segment 2	196	97	117	242	117	140	155
Segment 3	261	129	140	322	156	171	200
Segment 4	63	31	38	81	39	46	33
Segment 5	408	201	155	577	280	229	249
Segment 6	69	34	36	91	44	45	51
Segment 7	98	48	41	125	61	57	-14
Total	599	599	598	772	772	772	772
Sum of BUs	1,051			1,375			
Diversification Benefit	43.0%			43.9%			

Premium to Surplus						
VaR	Sc VaR	Eq Rs VaR	Sc TVaR	Eq Rk TVaR	Ph'ps TVaR
0.46	0.93	0.80	0.76	0.67	0.58
0.25	0.52	0.43	0.43	0.36	0.32
0.65	1.33	1.22	1.09	1.00	0.85
0.22	0.46	0.37	0.36	0.31	0.43
0.15	0.31	0.40	0.22	0.27	0.25
1.09	2.22	2.08	1.70	1.66	1.46
1.64	3.32	3.89	2.63	2.79	-11.79
0.98	0.98	0.98	0.76	0.76	0.76

Business Unit	TVaR Merton Perold	Shapley TVaR	Blended RAP	PH RAP	St Dev	Scaled St Dev	Cov / Beta
Segment 1	85	89	79	100	138	109	103
Segment 2	125	143	136	151	203	160	147
Segment 3	145	187	169	204	313	247	264
Segment 4	30	36	28	38	65	51	36
Segment 5	58	257	160	53	258	204	89
Segment 6	48	49	41	61	93	74	71
Segment 7	-16	11	-14	-8	144	114	62
Total	476	772	599	599	772	959	772

MP TVaR	Sh'ly TVaR	Blend RAP	PH RAP	Sc. SD	Beta
0.66	0.63	0.71	0.56	0.51	0.54
0.40	0.35	0.37	0.33	0.31	0.34
1.18	0.91	1.01	0.84	0.69	0.65
0.47	0.39	0.51	0.37	0.27	0.39
1.06	0.24	0.38	1.17	0.30	0.69
1.55	1.54	1.83	1.22	1.02	1.06
-10.23	13.91	-11.09	-20.88	1.41	2.59
1.23	0.76	0.98	0.98	0.61	0.76





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Why variance and standard deviation?

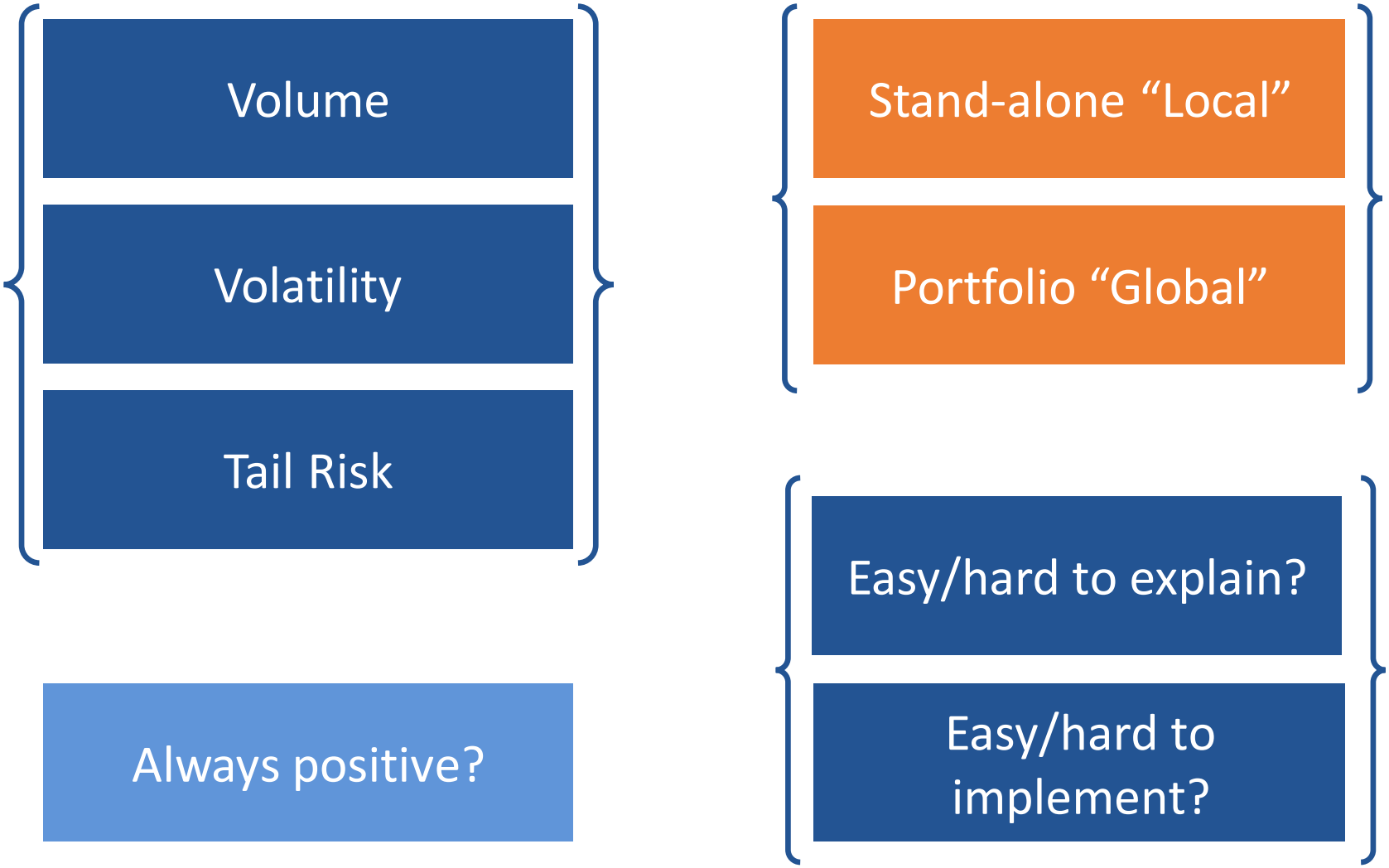
- Assumption: assume risk preferences are determined by mean and standard deviation of return
  - Securities market line, CAPM

- Utility, certain equivalent pricing c for a **small**, mean zero risk X
  - $U(w - c) = E[\, U(w - X)\,]$ which implies
  - $U(w) - c\, U'(w) = U(w) + Var(X)\, U''(w)/2$ and so
  - $c = -Var(X)/2\, U''(w) / U'(w)$, latter is called degree of absolute risk aversion

- In theory of Levy processes (=best model of insurance losses) variance corresponds to the continuous, no-jump part of the process

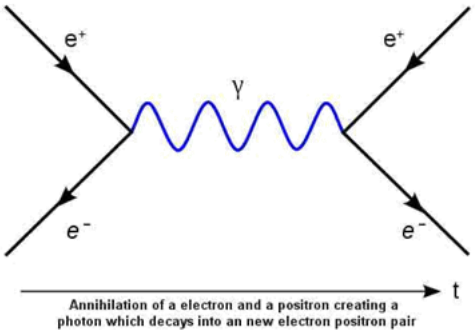
- Variance / standard deviation is not appropriate for larger jumps

Comparison of risk measures and allocations



A plethora of opinions; a paucity of facts

Accuracy of Predictions



Number of Opinions

Section 2: Practice

Knightian uncertainty: “We simply do not know!”

- “By “uncertain” knowledge … I do not mean merely to distinguish what is known for certain from what is only probable. The game of roulette is not subject, in this sense, to uncertainty … The sense in which I am using the term is that in which the prospect of a European war is uncertain, or the price of copper and the rate of interest twenty years hence, or the obsolescence of a new invention … **About these matters there is no scientific basis on which to form any calculable probability whatever. We simply do not know!**”
  - John Maynard Keynes, *General Theory of Employment, Interest and Money* (1936)

- What is the probability of tort reform in a given state?
  - What is the probability it will be over-turned?
- Chaos theory
- Maximum granularity driven by available information
- Michigan Catastrophic Claims Association…

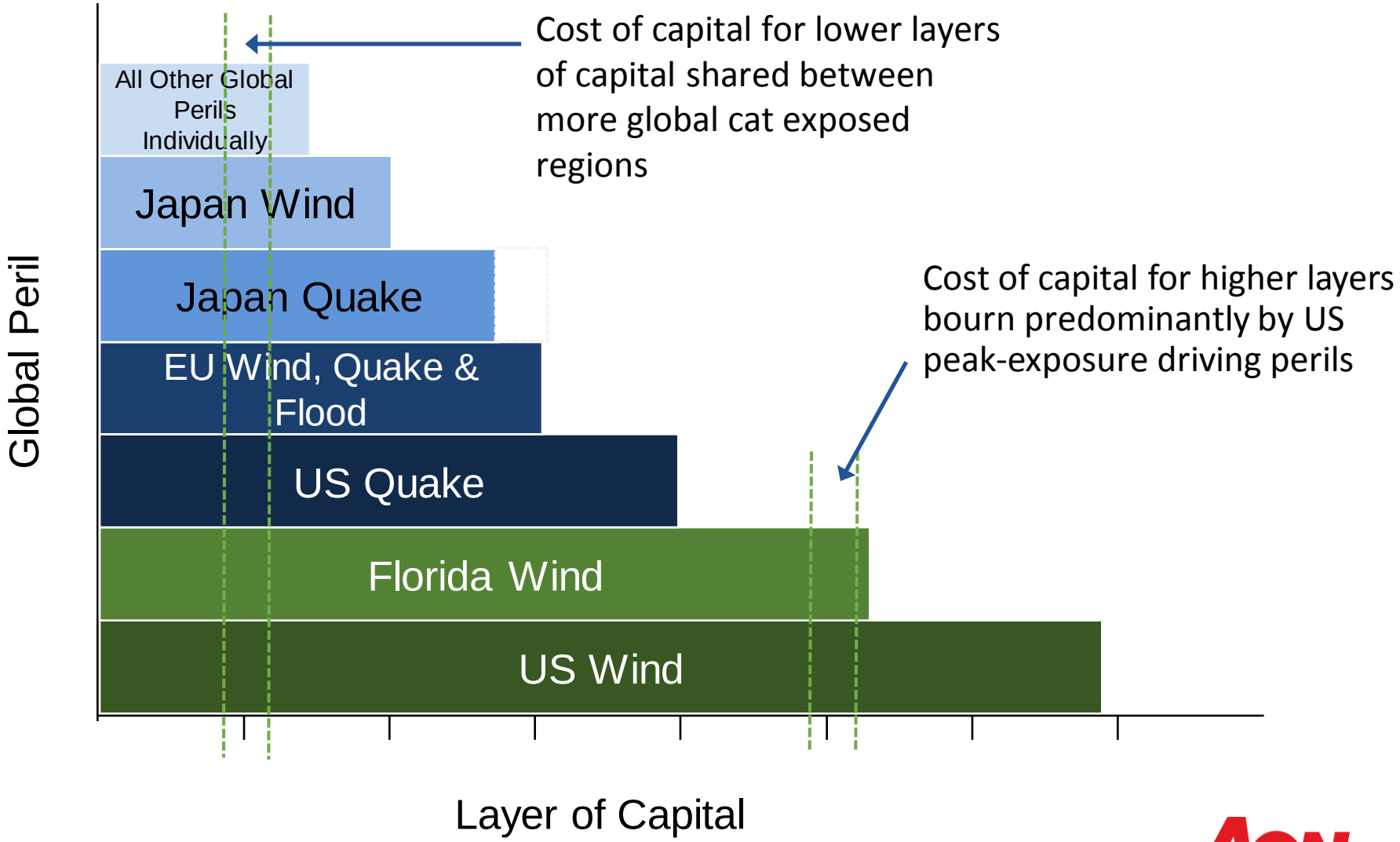
Where does capital allocation matter in practice?

Why do you say that?

- **For capital allocation to matter, profit margin must vary materially between insureds**
- For profit margin to vary materially it must be material
- Most lines of insurance are written with underwriting profit margins of 10% or less
- Pure cat risk produces significant premium at significant margins
- **Profit margin must also vary materially in a way that can be modeled**

Evidence from the real world

Global cost of catastrophe reinsurance capacity by layer



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Evidence from the real world considered

- Picture is descriptive of supply & demand, and regulatory realities
  - It shows the aggregate result of independent company actions
  - It is consistent with higher pricing in peak zones observed in the market
  - Explains macro pricing dynamics but lacks true predictive power at the company level
  - Size of bars is an **input** to global industry picture

- Picture does not solve an economic optimization problem for any agent
  - Pricing produced by individual optimization decisions, driven by risk measure and capital constrained optimization interacting with heterogeneous global distribution of risk
  - Company selection of limits and capacity deployment is a decision variable: individual company picture will not mirror industrywide picture
  - Size of the bars is a model **output** for individual companies

Our Cat Actuarial / Cat Score journey showed…

Gross Expected Loss

- Expected loss (AAL) is a function of policy terms and conditions but is the same estimate for any carrier
- Determined using catastrophe models at the risk and policy level

Reinsurance Margin

- Determined specifically based on **client program**
- Allocation driven by individual location's **contribution to loss ceded** to reinsurers and **correlation to industry loss**
- Calibrated using Aon Benfield database of observed industry pricing

Cost of Net Capital

- Calibrated to **client view** of capital required to support property cat and cost of capital
- Risk and capital allocation is determined by volatility and correlation with **client portfolio**

…pricing based on marginal VaR, TVaR did not agree with underwriter expectations of risk loads

Risk measures miss many important considerations

- Cost of acquiring new business or changing the portfolio
- Single year vs. multi-year view
  - Life time policyholder value concept used (talked about) in personal lines
- Unmodelable risk = social risk: driving forces dynamic, today's model not predictive tomorrow
- Unparameterizable risk = lack experience: three pandemics in last 100 years, non with modern travel patterns, populations, or medical technologies
- Capital: actually on balance sheet vs. available in market
  - Pre- and post-event funding, availability and cost; dilution
- I really care about shareholder value…

Loose linkage between risk and capital and valuation

2013 BCAR Scores	A++	A+	A	A-	B++	B+
25th percentile	250	234	231	208	166	146
75th percentile	322	412	415	439	284	246
Inter quartile Range	1.3	1.8	1.8	2.1	1.7	1.7

Value to Risk "flexibility" of at least 4:1

Source: A.M. Best's data dated June 30, 2014 and Aon Benfield Analytics

The current state of play and why it makes sense…

- It is no surprise we see global convergence towards simple factor based models for measuring risk for non-cat lines combined with more sophisticated model-driven assessment of cat risk
  - Standard formula in S2
  - RBC with revised cat load
  - BCAR
  - S&P CAR
  - Factors driven by more (=Insurance Risk Study) or less rigor

- Model shortcomings largely recognized by users
  - Operational risk charge =10 to 25% surcharge
  - Events not in experience period excluded; models extend the experience period

Recap

Viewpoint	Property Cat	Casualty
Quantitative	**Process risk world** $60B loss / yr keeps current Clear capital driver Physical basis for model Physical basis for correlation Tested models Clear optimization approach	Small risk, standard severity understood: commercial auto, work comp Pricing cycle understood Slippage in terms & conditions
Qualitative	Model miss Warm SST impact ENSO impact Data quality	**Knightian uncertainty** Time uncertainty: development Self-regulating social system?
Risk Measure / Capital Adequacy	VaR or TVaR used in almost all adequacy models AMB, SP, RBC, S2, SST	Unclear Factor based consensus
Capital Allocation	Standard marginal approach	Not practically possible